we have $u_{n2} > c_2$. For the values of q satisfying the inequality (4) the conditions of evolutionarity hold on the segment CC_1 of the adiabatic curve. When the inequality $2p_1 + q(\gamma - 1) < 0$ holds, the function $p_2(v_2)$ increases monotonously from $-\infty$ at $v_2 = v_1 / \varkappa$ to $p_2 = -(p_1 - q) / \varkappa$ when v_2 tends to ∞ (Fig. 3). Thus, the whole of the curve $p_2(v_2)$ lies below the axis $p = 0$ and has no physical meaning.

REFERENCES

1. Sedov, L. I., Mechanics of Continuum. Ed.2, Moscow, "Nauka", 1973.
2. Landau, L. D. and Lifshits, E. M., Electrodynamics of Continuous Media. (English translation), Pergamon Press, Book № 09105, 1960.
3. Tarapov, I. E., On the basic equations and problems of hydrodynamics of polarizable and magnetizable media. In the book: Theory of Functions, Functional Analysis and their Applications, № 17, Izd. Kharkov. Univ., 1973.

Translated by L. K.

UDC 532.5

LONGITUDINAL FLOW PAST A SLENDER BODY OF REVOLUTION WITH A FREE BOUNDARY

PMM Vol. 39, № 1, 1975, pp. 185-188
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(Received June 2, 1972)

We investigate a longitudinal flow past an axisymmetric body in the case when a part of the streamlined surface is not known but instead, the distribution of the tangential velocities is specified. The flow is assumed irrotational, and the fluid ideal and incompressible. At the stagnation points the body surface may behave as a sphere, a cone or an edge. An integro-differential equation for determining the form of the free surface is derived for any arbitrarily specified velocity. In the case of a cavitation flow the method of the undetermined coefficients is used to solve the above equation. An analytic and graphical dependence of the cavitation number on the apex angle of cone and its relative length, is given. The theory is satisfactorily confirmed by experimental data.

1. Statement of the problem. Let a longitudinal irrotational stream of an ideal incompressible fluid flow past a slender axisymmetric body. The surface of this body is described by the equation $\rho = R(z)$, where

$$R(z) = \begin{cases} r_-(z) & -1 < z < b \\ r(z) \quad \text{as given by the condition} \quad v_\tau = v_\tau(z), & b < z < c \\ r_+(z) & c < z < 1 \end{cases} \qquad (1.1)$$

The segment (b, c) which is defined by the distribution of tangential velocities $v_\tau(z)$ is a free boundary, while the segments $(-1, b)$ and $(c, 1)$ are parts of the rigid boundary.

The problem is reduced to finding the equation of the free boundary $r(z)$. We assume that the streamlined surface satisfies the following conditions:

1) functions $R^2(z)$ and $dR^2(z)\,/\,dz$ are continuous and $R(-1) = R\,(1) = 0$,

2) function $d^2R^2(z)\,/\,dz^2$ is piecewise continuous and has first order discontinuities at the points b and c,

3) $R^2(z) < \varepsilon, \left|\dfrac{d^k}{dz^k} R^2(z)\right| < \varepsilon,\ k = 1,2,3$ (conditions of thinness).

The conditions under which the equation of the free boundary $r\,(z)$ satisfies the above restrictions, have been obtained in [1] (except the inequality for the third order derivative which we introduce here in order to simplify certain expressions).

For a sufficiently small ε the potential of the longitudinal flow past such a body (the velocity of the unperturbed flow is taken as unity) is given by the following approximate formula [2]:

$$\varphi\,(z, \rho) = z + \int_{-1}^{1} P\,(\zeta)\,\frac{z - \zeta}{[(z - \zeta)^2 + \rho^2]^{3/2}}\,d\zeta + \Gamma\,(z, \rho) \tag{1.2}$$

$$P\,(\zeta) = \frac{1}{4}\,R^2\,(\zeta)\,[1 - \gamma\,(\zeta)], \quad \gamma\,(\zeta) = \frac{1}{2}\sum_{i=1}^{2} \frac{\rho_i^3}{[(\zeta - c_i)^2 + R^2\,(\zeta)]^{3/2}} \tag{1.3}$$

where ρ_1 and ρ_2 are the radii of curvature of the surface at the stagnation points. The functions Γ and γ account for the influence of rounding at the stagnation points.

Let us integrate (1.2) by parts; we set $\rho = r\,(z)$, differentiate with respect to z and again integrate by parts

$$\int_{-1}^{1} P''\,(\zeta)\,\frac{d\zeta}{\sqrt{(\zeta - z)^2 + p}} - \frac{1}{2}\,\frac{p'}{p}\int_{-1}^{1} P'\,(\zeta)\,\frac{p}{[(\zeta - z)^2 + p]^{3/2}}\,d\zeta + [\varphi'\,(z) - 1] = \Gamma'(z) \tag{1.4}$$

Here $p = r^2\,(z)$, $\varphi\,(z) = \varphi\,(z, r, (z))$ and $\Gamma\,(z) = \Gamma\,(z, r, (z))$.

We note that the functions $\Gamma\,(z)$ and $\gamma\,(\zeta)$ are of the order of ε^3 everywhere except at the stagnation points, and can therefore be almost always neglected (the integrals are of the order of ε). Their influence will only become noticeable in the case of flows past spheres with free boundaries when the flow separation point has to be determined. From this it follows that on the segment (b, c) we can assume $P\,(\zeta) = {}^1\!/_4\,p\,(\zeta)$ which simplifies the computations considerably.

Thus, the equation of the free surface $\rho = r\,(z)$ can be found using the relation $r\,(z) = \sqrt{p\,(z)}$ from the nonlinear integro-differential equation (1.4) in which the function $\varphi'\,(z)$ will be determined later.

2. Basic equation. In order to simplify (1.4) we shall determine the order of smallness of each term in its left-hand side. Using the relations

$$i\,(z) = \int_{-1}^{b} P_-''\,(\zeta)\,[(\zeta - z)^2 + p]^{-1/2}\,d\zeta + \int_{c}^{1} P_+''\,(\zeta)\,[(\zeta - z)^2 + p]^{-1/2}\,d\zeta \tag{2.1}$$

$$\lambda\,(z) = P''\,(b)\ln\frac{|b - z| + \sqrt{(b - z)^2 + p}}{2} + P''\,(c)\ln\frac{c - z + \sqrt{(c - z)^2 + p}}{2} \tag{2.2}$$

$$\ln\frac{x+\sqrt{x^2+p}}{2}=\begin{cases}\ln\dfrac{p}{4}-\ln\dfrac{|x|+\sqrt{x^2+p}}{2}, & x<0\\ \ln\dfrac{x+\sqrt{x^2+p}}{2}, & x>0\end{cases}$$

we can rewrite the first term in the form

$$\int_{-1}^{1}\frac{P''(\zeta)\,d\zeta}{\sqrt{(\zeta-z)^2+p}}=-P''(z)\ln\frac{p}{4}+i(z)+\lambda(z)- \qquad (2.3)$$

$$\int_b^c P'''(\zeta)\operatorname{sgn}(\zeta-z)\ln\frac{|\zeta-z|+\sqrt{(\zeta-z)^2+p}}{2}\,d\zeta$$

The second term of (1.4) can be written, under the assumption that the body is slender, in the form [2]

$$\frac{1}{2}\int_{-1}^{1}P'(\zeta)\frac{p\,d\zeta}{[(\zeta-z)^2+p]^{3/2}}=P'(z)\,[1+O(p\ln p)]$$

and for the last remaining term we have

$$\varphi'(z)==v_\tau\sqrt{1+r'^2}=v_\tau+{}^1/_2v_\tau r'^2$$

Thus, the basic relation (1.4) can be written in the form of a sum in which the terms are clearly arranged according to their order of smallness, with the accuracy of up to $p^2\ln p$ (we neglect p under the radical sign in the right-hand side of (2.3))

$$-P''(z)\ln\frac{p(z)}{4}-\int_b^c P'''(\zeta)\operatorname{sgn}(\zeta-z)\ln|\zeta-z|\,d\zeta+$$

$$i(z)+\lambda(z)-\frac{P'(z)\,p'(z)}{p(z)}+\frac{1}{8}v_\tau\frac{p'^2(z)}{p(z)}+v_\tau-1=\Gamma'(z)$$

The above basic equation enables us to find the equation of the free boundary $r(z)=\sqrt{p(z)}$ under the condition that the tangential velocity is equal to v_τ. Here $P(z)={}^1/_4\,p(z)\,[(1-\gamma)(z)]$ and the functions $i(z)$, $\lambda(z)$, $\Gamma(z)$ and $\gamma(z)$ are defined by (1.4), (2.1) and (2.2).

3. Cavitation flow. We shall assume the velocity at the free boundary to be constant $v_\tau=\sqrt{1+Q}$ (Q is the cavitation number), so that at small Q we have $v_\tau=1+{}^1/_2\,Q$. Neglecting the influence of the stagnation points ($\Gamma=0$, $\gamma=0$), we obtain

$$-p''(z)\ln\frac{p(z)}{4}-\int_b^c p'''(\zeta)\operatorname{sgn}(\zeta-z)\ln|\zeta-z|\,d\zeta+ \qquad (3.1)$$

$$\bar{i}(z)+\bar{\lambda}(z)-\frac{1}{2}\frac{p'^2}{p}+2Q=0$$

$$\bar{i}(z)=\int_{-1}^{b}p_-''(\zeta)\,[(\zeta-z)^2+p]^{-1/2}d\zeta+\int_c^1 p_+''(\zeta)\,[(\zeta-z)^2+p]^{-1/2}d\zeta$$

$$\bar{\lambda}(z)=p''(b)\ln\frac{|b-z|+\sqrt{(b-z)^2+p}}{2}+p''(c)\ln\frac{c-z+\sqrt{(c-z)^2+p}}{2}$$

The relation (3.1) allows us to find the form of the streamlined free surface. If we assume that $|\ln p| \gg 1$, so that the function $\lambda(z)$ and the definite integral in (2.3) can both be neglected, we arrive at the equation obtained in [3].

Equation (3.1) can be solved using the method of undetermined coefficients. Let us set $p(z) = a_0 + a_1 z + \ldots + a_n z^n$, and require that the basic equation (3.1) holds at the nodes $z_1, z_2, \ldots, z_{n+1}$. This yields a system of $n+1$ equations with $n+1$ unknowns $a_0, a_1, \ldots, a_n$, and by determining these unknowns we obtain the equation $r(z) = \sqrt{p(z)}$ of the cavity.

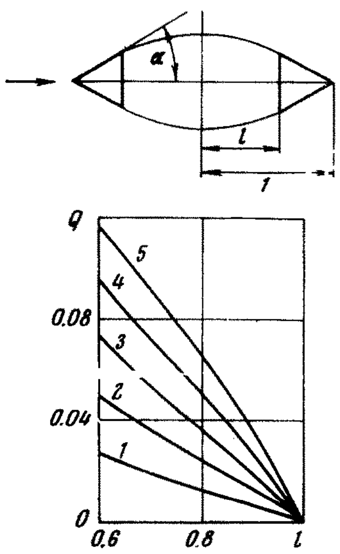

Fig. 1

The solution simplifies considerably in the case of a Riabushinskii flow (symmetrical relative to the plane $z = 0$), when only even powers of z remain in the expansion. In this case we can utilize the expression

$$p(z) = p(l) + \frac{p'(l)}{2l}(z^2 - l^2) + \sum_{k=2}^{n} b_k (z^2 - l^2)^k$$

which ensures that the specified value of function $p(z)$ and of its derivative at the trailing and leading edges ($z = \pm l$) is achieved.

We illustrate the proposed theory by considering a cavitation flow past a cone in accordance with the Riabushinskii model. We take the cavitation number Q as the free parameter and find its dependence on the half-length of the cavity l and on the tangent of the apex half-angle of cone $k = \operatorname{tg}\alpha$. The equation (3.1) with $z = 0$ was used to compute Q. The results are shown in Fig. 1. The solid curves $1-5$ correspond to the following values of k : 0.1, 0.15, 0.2, 0.25, 0.3. If l is nearly equal to unity, which corresponds to the case of a long (compared to the cone) cavity, then the following formula is recommended:

$$Q = k^2 b \ln\frac{0.4}{k^2 b}, \quad b = \frac{1-l}{l}, \quad k \leqslant 0.3$$

For $k = 0.268$ (apex angle 30°) the cavitation numbers computed according to the above formula, deviate from the experimentally obtained data [4] only by 4% (provided that the length of the real cavity is $1 + l$).

REFERENCES

1. Taits, O. G., Flow past a slender axisymmetric body with velocity distribution given on a part of the surface. Zh. prikl. mekhan. i tekhn. fiz., №1, 1968.
2. Taits, O. G., Motion of a slender axisymmetric body in an incompressible fluid at an angle of attack. Vestn. LGU, Leningrad, №13, 1965.
3. Iakimov, Iu. L., Axisymmetric detached flow past a solid of revolution with small cavitation numbers. PMM Vol. 32, №3, 1968.
4. Epshtein, L. A., Bliumin, V. I. and Fediushin, P. A., Experimental estimation of the entrainment of gas and the cavity length behind cones. Tr. TsAGI, №1100, 1968.

Translated by L.K.